Misconceptions in Numbers

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Misconceptions associated with numbers are found throughout the mathematics curriculum. Here, we identify and review certain misconceptions that are most common among primary and secondary school students.

1. Misconceptions Associated with the Arithmetic Operations

Of the four basic arithmetic operations *addition* seems to present students with the least challenges. Two of the most common errors relate to the positioning of the numbers in the vertical presentation of the addition and the process of 'carrying'. Both of these errors are symptomatic of a lack of understanding of place value.

In *subtraction*, Dickson & al (1984) cite Resnick (1982) summary of student's most common errors as either Smaller-from-Larger or mistakes with borrowing. Smaller-from-Larger refers to the fact that students would take the smaller digit from the larger irrespective of the position of the digits as in the following example

$$\begin{array}{r} 543 \\ -\ 237 \\ \hline = \quad 314 \end{array}$$

Clearly, the student subtracted 3 from 7 because 3 was the smaller digit. Here, the assumption is that the subtraction is commutative. An explicit reference to the non-commutativity of subtraction will reduce the occurrence of such errors. Students should be made aware at an early age of the importance of order in subtraction.

Of the four arithmetic operations, *division* presents students with the most challenges. The first is the belief that the divisor should always be smaller than the dividend. The extent of this problem was investigated by several researchers. Graeber and Baker (1992) put the following question to fifteen nine-year old and fifteen ten-year old children:

> *"Five pounds of trail mix was shared equally by fifteen friends. How many pounds of trail mix did each friend get?"*

Twenty-four out of the group of 30 children responded by performing the operation 15÷5 and giving 3 as the answer. Perhaps more worrying was the fact that 42 percent of a sample of sixty-five trainee elementary school teacher trainees gave the answer 15÷5.

The source of this misconception lies clearly in the students' early encounters with division. Such encounters are almost always in situations where a whole number has to be divided by one of its factors. Thus, for the children, 'this kind of problem always goes like that, the little into the big.' Further research (Graber and Baker 1988) suggests that throughout KS2 and KS3 students encounter very few instances where the divisor is greater than the dividend.

2. Misconceptions With Zero

There is a wide range of common errors that students make when they encounter zero in arithmetic operations. Perhaps the most common is the problem that students have in 'borrowing' from zero in the process of subtraction. The use of zero in multiplication and division is also the source of a large number of mistakes and misconceptions among students of all ages.

Multiplying by zero

One of the most common mistakes involving zero is the failure by many students to realise that multiplying any number by zero yields zero. Rees and Barr (1984) found that 52% of 8613 people in a public examination wrote that

$$9 \times 0 \times 8 = 72$$

This failure stems probably from the difficulty that many students have in interpreting a multiplication by zero. For many, zero represents nothing. As such, a multiplication by zero just leaves the number unchanged.

Is Zero Worth Writing?

Often students are confused when trying to decide whether to write or omit zero. They are often told that zero at the end of a decimal number has no value and therefore can be omitted without changing the number. Thus

45.80 is identical to 45.8

Similarly, when dividing 1632 by 8, students are taught not to write the '0' that 3 divided by 8 would yield and divide 16 by 8 instead. As a result, many students become confused and are unable to determine exactly when should zero be written and when it should be omitted. Thus, students often make mistakes of the type:

$$8\overline{)1632}^{\;24}$$

It is not difficult to see the rationale behind the result. Since 8 does not divide into 3, students often move on to divide 8 into 32 to obtain 4. Thus the zero that should have been written between 2 and 4 is omitted, resulting in 24 instead of 204.

3. Decimals

More students have problems with *decimals* than with any other number concept. There seems to be a large gap between students' understanding of natural numbers and their understanding of decimal numbers. It's 'as if the introduction of the decimal point changes the nature of the number in a fundamental way'. Brown (1981b, 1981c) found that about half of 12 year-old students and a third of 15 year olds have difficulties understanding decimal notation.

Difficulties with decimal numbers range from comprehending place value after the decimal to proper use of the algorithm of addition and subtraction. Rees and Barr (1984) found that 10 year-old primary school students provided *100 different answers* to the following task:

16.36 + 1.9 + 243.075

The most common (wrong) methods used by students when adding decimals include:

- Adding the numbers before the decimal points and the numbers after the decimal points separately and combining them in any one of a number of ways to form a single number.
- Mistakes in aligning the numbers vertically and using the addition algorithm.

Multiplication and division of decimal numbers are even more challenging to students. Common misconceptions often result from a lack of feel of multiplication or division by decimals.

It Gets Bigger When I Multiply and Smaller When I Divide

Many problems with decimal numbers stem from facts learned or perceived when working with natural numbers. Many properties of natural numbers are mechanically extended to other real numbers, leading to many erroneous beliefs. Prominent among these is the widespread belief that multiplication should always yield a number that is necessarily higher than those with which we started and a division should result in a smaller number.

Thus for many students it seems inconceivable that 5×0.4 should give 2 since 2 is smaller than 5. In the same way, they find it hard to accept that $10 \div 0.1$ gives 100 since 100 is much bigger than 10? For many children, to make a number bigger, you have to multiply it and to make it smaller you have to divide it.

Why do many students believe that multiplication makes bigger? The reasons are multiple. First, many everyday language expressions imply that it is the case. When plants or animal reproduce we talk of multiplication. The word ‘multiple’ itself carries a sense of many or a great number. The second reason for this misconception has to do with the fact that many students first encounter multiplication in the context of whole numbers, situation in which the multiplication of two numbers indeed results in a larger number. A third reason for this misconception is suggested by Graeber and Campbell (1993). Multiplication is often explained to young students as a repeated addition. This view carries many secondary effects of which the most prominent is that multiplication makes bigger. Other secondary effects include the difficulty that students have in dealing with multiplication of fractions, and dealing with multiplication with small numbers.

4. Fractions

Kerslake (1986) found that students of thirteen to fourteen years relied heavily on rote memory of previously learned techniques when working with fractions. She believes that this is mainly due to the fact that “fractions do not form a normal part of a child’s environment and the operations on them are abstractly defined”. This abstraction and lack of feel for fractions lead students to have many misconceptions about fractions. The most common of these misconceptions involve the four algebraic operations and equivalent fractions.

Multiplication and Division of Fractions

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Research shows that on the whole students cope well with multiplication of fractions. This is probably due to the fact that the rule of multiplication of fractions seems natural: multiply the numerators together and the denominators together. Cramer and Bezuk (1991) found that even lower-ability students experience little or no difficulty in multiplying fractions. However, the same research also concluded that students have little or understanding of the concept of multiplication of fractions. This is partly due to the fact that multiplication with whole numbers is often viewed and understood by students as a repeated addition. When confronted with an operation of the type

2/3 × 3/5

$$\frac{2}{3} \times \frac{3}{5}$$

students have no meaningful interpretation.

In view of this absence of interpretation, efforts to teach multiplication of fractions should focus more on real situations involving a product of two fractions and the explication of why the multiplication of the two fractions is performed the way it is.

Division of fractions is even more problematic. Students find it extremely difficult to concretely visualise a division of say, ¾ by ½. For most children, this kind of operation is simply meaningless. In addition, often students fail to see the point or understand the logic behind the inversion of the second fraction when carrying the operation.

Addition and Subtraction of Fractions

Addition of fractions poses a different sort of problem to children. While most will have no problems comprehending the meaning of ¼ + 2/3, often they will struggle to find the right result. When faced with the addition of fractions, often students choose the "easiest way out". Instead of looking for equivalent fractions having the same denominator, they simply add the numerators and the denominators, thus using the following "rule":

$$\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$$

Hart (1981) found that 30% of 13 year olds were making this error, and notes that 15 year olds were almost as likely to make this error as 13 year olds. This misconception is, at least partially, a result of the rule of multiplication of fractions.

Equivalent Fractions

The concept of equivalent fractions is needed in many applications involving fractions. For example, if we wanted to know which is the larger of 3/5 and 4/7, one way of dealing with this would be to find fractions equivalent to the given ones, but having the same denominator. However, many students of all ages experience difficulties in their attempt to find equivalent fractions. Hart (1980) found that only 66 percent of 15 year olds could recognize that 3/10 was larger than 1/5. In an American Survey (NAEP), only 3 percent of 13 year olds were to finds which of the fractions 1/4, 5/32, 5/16, 3/8 was nearest to 3/16. These results clearly demonstrate that students either do not know how to find equivalent fractions or do not make the connection between equivalence and size.

Equivalence of fractions can also be used to find a fraction between two given fractions, such as 1/2 and 2/3. Hart (1980) found that only 21 percent of 15 year olds were able to find such a fraction. Hart also found that students often do not realise that between two fractions on the number line there are many (infinitely many) fractions. The following table summarises the replies given by 15 year olds to the question:

'How many fractions lie between ¼ and ½?'

Response	Percentage of children
Infinitely many, lots, etc… (correct answer)	16%
One	30%
A number between 1 and 20	22%
Other answers	15%
Omit	17%

Brown (1981) confirms that students often do not know that the number line has no 'empty spaces' when he asked students the question: 'How many different numbers could you write down which lie between 0.41 and 0.42?'

5. Percentages.

Percentages, ratios and proportions present children, and indeed most people, with a number of challenges that appear quite daunting. Rees and Barr (1984) found that only half of a sample of 8600 candidates could work out their new salary if their present salary increased by a given percentage. In another test, only 26% of twelve-year old students could work out how much a pair of jeans which normally costs £15 would cost after a 20% reduction. Thirty-three different answers were given to the question. The APU (1980) reports that only about half of 15 year olds could work out what percentage 50 is of 250.

Here is a sample of some answers given to a simple percentage question:

20% of £65 = £ 65/20
20% of £65 = £ 65/20 × 100
20% 0f £65 = 15%

The wide range of answers shows that there is widespread confusion linked to percentages. Any lesson on percentages should start by a clear understanding that a percentage is basically a special fraction or a decimal.

CONCLUSION

Misconceptions abound in mathematics. They are picked throughout the educational life of a child. Some are inherent to the subject, others are the results of teaching techniques that encourage the emergence of such misconceptions. Researchers agree that most misconceptions are difficult to overcome. Thus it is more important for teachers to make sure that the misconceptions do not arise in the first place. First, teachers should be aware of areas that have the potential to generate misconceptions in the minds of the children. Then enough work, and examples should be focused on directly contradicting the perceived misconception.

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